Factorising quadratics

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You will have seen before that expressions like $(x + 2)(x + 3)$ can be expanded to give the quadratic expression $x^2 + 5x + 6$. Like many processes in mathematics, it is useful to be able to go the other way. That is, starting with the quadratic expression $x^2 + 5x + 6$, can we carry out a process which will result in the form $(x + 2)(x + 3)$? This process is called **factorising the quadratic expression**. This leaflet describes this process. Special cases known as **complete squares** and **the difference of two squares** are dealt with on separate leaflets.

Factorising quadratics

To learn how to factorise let us study again the previous example when the brackets were multiplied out from $(x + 2)(x + 3)$ to give $x^2 + 5x + 6$.

$$(x + 2)(x + 3) \quad = x^2 + 3x \quad + \quad 2x + 6 \quad = x^2 + 5x \; + \; 6$$

Clearly the number 6 in the final answer comes from *multiplying* the numbers 2 and 3 in the brackets. This is an important observation. The term $5x$ comes from *adding* the terms $3x$ and $2x$.

So, if we were to begin with $x^2 + 5x + 6$ and we were going to reverse the process we need to look for two numbers which add to give 5 and multiply to give 6. What are these numbers ? Well, we know that they are 3 and 2, and you will learn with practice to find these simply by inspection. We can set the calculation out as follows. Start with a pair of empty brackets.

$$\begin{aligned} x^2 + 5x + 6 &= (\quad\quad)(\quad\quad) && \text{insert an } x \text{ in each} \\ &= (x \quad\quad)(x \quad\quad) && \text{these will multiply to give the required } x^2 \\ &= (x + 2)(x + 3) && \text{these numbers multiply to give 6 and add to give 5} \end{aligned}$$

The answer should always be checked by multiplying-out the brackets again!

Example

Factorise the quadratic expression $x^2 - 7x + 12$.

Starting as before we write

$$x^2 - 7x + 12 = (x \quad\quad)(x \quad\quad)$$

and we look for two numbers which add together to give -7 and which multiply together to give 12. The two numbers we seek are -3 and -4 because

$$-3 \times -4 = 12, \qquad \text{and} \qquad -3 + -4 = -7$$

So

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Once again, note that the answer can be checked by multiplying-out the brackets again. The alternative, equivalent form $(x - 4)(x - 3)$, is also correct.

Exercises

1. Factorise the following.

a)	$x^2+8x+15$	b)	$x^2+10x+24$	c)	x^2+9x+8	d)	$x^2+9x+14$
e)	$x^2+15x+36$	f)	x^2+2x-3	g)	x^2+2x-8	h)	x^2+x-20

Quadratic expressions where the coefficient of x is not 1

Let us try to factorise the expression $3x^2+5x-2$. We write, as before,

$$3x^2+5x-2 = (\qquad)(\qquad)$$

and try, by inspection, to determine the contents of the brackets. There is no point writing $(x\qquad)(x\qquad)$ because the two x terms would multiply to give x^2, and in this example we are looking for $3x^2$. So try

$$3x^2+5x-2 = (3x\qquad)(x\qquad)$$

which will certainly generate the term $3x^2$. The constant term -2 can be generated from the numbers -2 and 1, or alternatively -1 and 2. So, we are led to consider the following combinations

$$(3x-2)(x+1),\qquad (3x+1)(x-2),\qquad (3x-1)(x+2),\qquad (3x+2)(x-1)$$

all of which generate the correct term in x^2 and the correct constant term. However, only one of these generates the correct x term, $5x$. By inspection we find

$$3x^2+5x-2 = (3x-1)(x+2)$$

Example

Factorise $2x^2+5x-7$.

To generate the term $2x^2$ we can write

$$2x^2+5x-7 = (2x\qquad)(x\qquad)$$

To generate the constant term -7 we need two numbers which multiply together to give -7. Recognise that to produce a negative result one factor must be positive and one must be negative. We are led to consider -7 and 1, or alternatively -1 and 7. So, we consider the following combinations

$$(2x-7)(x+1),\qquad (2x+1)(x-7),\qquad (2x-1)(x+7),\qquad (2x+7)(x-1)$$

By inspection the correct factorisation is $2x^2+5x-7=(2x+7)(x-1)$.

Exercises

2 Factorise the following.

a)	$2x^2+11x+5$	b)	$3x^2+19x+6$	c)	$3x^2+17x-6$	d)	$6x^2+7x+2$
e)	$7x^2-6x-1$	f)	$12x^2+7x+1$	g)	$8x^2+6x+1$	h)	$8x^2-6x+1$

Answers

1.	a)	$(x+3)(x+5)$	b)	$(x+4)(x+6)$	c)	$(x+1)(x+8)$	d)	$(x+2)(x+7)$
	e)	$(x+3)(x+12)$	f)	$(x+3)(x-1)$	g)	$(x+4)(x-2)$	h)	$(x+5)(x-4)$
2.	a)	$(2x+1)(x+5)$	b)	$(3x+1)(x+6)$	c)	$(3x-1)(x+6)$	d)	$(2x+1)(3x+2)$
	e)	$(7x+1)(x-1)$	f)	$(3x+1)(4x+1)$	g)	$(2x+1)(4x+1)$	h)	$(2x-1)(4x-1)$

